



Junior Analysis Seminars 2026

October 6th, 2026 – 3 pm

Scuola Superiore Meridionale - via Mezzocannone, 4
Room: Aula 2

The Distributional Laplacian of the Distance Function and Warped Product Splitting

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Abstract. In 2003, Mantegazza and Mennucci proved that the cut locus of a closed embedded C^r submanifold, with $r \geq 3$, of a complete Riemannian manifold (M^n, g) is $(n - 1)$ -rectifiable. In 2014, Mantegazza, Mascellani, and Uraltsev used this rectifiability result to derive a global representation of the distributional Laplacian of the distance function $d_p = d_g(p, \cdot)$ from a point $p \in M$. For $n \geq 2$, the following identity holds as an equality of Radon measures on M :

$$\Delta_g^{\text{dist}} d_p = (\Delta_g^{\text{cl}} d_p) \text{Vol}_g - |(\nabla d_p)^+ - (\nabla d_p)^-|_g (\mathcal{H}_g^{n-1} \text{Cut}(p)).$$

Here, Δ_g^{dist} and Δ_g^{cl} denote the distributional and classical Laplacians, respectively, and Vol_g is the Riemannian volume measure. The singular term is concentrated on the cut locus $\text{Cut}(p)$ of the point p and involves the one-sided approximate traces $(\nabla d_p)^\pm$ of the gradient. The notation $\mathcal{H}_g^{n-1} \text{Cut}(p)$ denotes the $(n - 1)$ -dimensional Hausdorff measure restricted to $\text{Cut}(p)$.

In this seminar, we extend this description to the distance function from a sufficiently regular embedded submanifold and apply it to Riemannian bands. These are compact Riemannian manifolds whose boundary consists of two disjoint hypersurfaces, as exemplified by the region between two concentric geodesic spheres. Under the assumption of nonnegative Ricci curvature, we use this description to estimate the width of such a band and characterize the corresponding rigidity case.

Organizers. Francesca de Giovanni, Giuliana Fusco, Miriam Piccirillo

This seminar series is funded by Scuola Superiore Meridionale