Hadamard's inequality in the mean

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Let Q be a Lipschitz domain in $\mathbb{R}^n$ and let $f \in L^\infty(Q)$. We investigate conditions under which the functional

$$I(\varphi) = \int_Q |\nabla\varphi|^n + f(x) \det \nabla\varphi \, dx$$

obeys $I(\varphi) \geq 0$ for all $\varphi \in W_0^{1,n}(Q, \mathbb{R}^n)$. We prove that there are f such that $I \geq 0$ holds and is strictly stronger than the best possible inequality that can be derived using Hadamard's pointwise inequality $n^{n/2}|\det A| \leq |A|^n$ alone. Almost all of the f we consider are piecewise continuous, and we specialize to the case $n = 2$ in many occasions. We find that it is both the geometry of the 'jump sets' as well as the sizes of the 'jumps' themselves that determine whether $I \geq 0$ holds. We also outline connections to quasiconvexity at the boundary, regularity of minimizers, sequential weak lower semicontinuity, and to Agmon's conditions in elasticity. Theoretical results will be complemented with various numerical experiments. This is a joint work with J. Bevan (Surrey) and J. Valdman (Prague).