

# Avviso Seminario

*Dipartimento di Matematica e Applicazioni "Renato Caccioppoli"*  
*Università degli Studi di Napoli "Federico II"*

Lunedì 20 ottobre

Sala Riunioni 1L, h. 16:00

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*University of Tübingen*

*"Epsilon-theorems, cut-elimination, normalisation: from proofs as objects to the semantics of proofs"*

Abstract: The idea that proofs can be understood as objects is famously due to David Hilbert. The new mathematical field which stemmed from this idea is nowadays a well-established sub-field of formal logic called *Proof Theory*. Based on a distinction between "semantically stable" and "semantically unstable" mathematics, Hilbert aimed to use proof theory for the foundational purpose of proving mathematics itself to be consistent (i.e., non-contradictory). But Hilbert's program extends well beyond that. Through his first and second epsilon-theorem (proved with Paul Bernays), Hilbert made clear that the mathematical treatment of proofs-as-objects allowed for a kind of proof-manipulation that triggers more general considerations on the *structure* of proofs. Through this insight, a student of Hilbert, Gerhard Gentzen, introduced a new kind of proof theory, now called *general proof theory*. Besides some crucial structural results such as the cut-elimination theorem for his own *Sequent Calculus*, however, Gentzen also showed that this broader kind of proof theory allowed for the achievement of the first major goal of Hilbert's program, namely, a proof of the consistency of Peano (first-order) Arithmetic. Some thirty years later, Dag Prawitz expanded Gentzen's general proof theory even further, by proving certain normalisation results for the other kind of calculus invented by Gentzen, i.e., *Natural Deduction*. Prawitz's program is still partly foundational, as his normalisation theorems allow for a relatively simpler proof of the consistency of Peano (first-order) Arithmetic (as well as a proof of Takeuti's conjecture, which implies the consistency of Real and Complex Analysis). However, the main point of Prawitz's theorems is that they completely unfold the true content of Gentzen's turn: once proofs are understood as objects, it is very natural to develop a formal study of the semantics of these objects. Indeed, Prawitz's normalisation theorems lie at the core of *Proof-Theoretic Semantics*, a major formal semantics of our days (also due to Prawitz) that I will outline at the end of my talk.

La Proponente  
Giuseppina Terzo