Limit points of A_α-matrices of graphs

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Abstract

In 1972, A. J. Hoffman introduced the concept of limit points of eigenvalues of graphs. Let $\mathcal{A}$ be the set of all symmetric matrices of all orders, in which every entry is a natural number, and $R = \{\rho : \rho = \rho(A) \text{ for some } A \in \mathcal{A}\}$ where $\rho(A)$ is the largest eigenvalue of A. Hoffman asked which real numbers can be limit points of R , showed that it is sufficient to consider matrices of $\mathcal{A}$ having entries in $\{0, 1\}$ and 0 diagonal, e.g. adjacency matrices of graphs, and determined all limit points of $R \leq \sqrt{2+\sqrt{5}}$. In 1989, a remarkable result due to J. B. Shearer extended the work of Hoffman. He showed that every real number larger than $\sqrt{2+\sqrt{5}}$ is a limit point of R. There is a considerable amount of work, extending the results to other matrices related to graphs, as well as to eigenvalues other than the spectral radius.

In this talk, we are interested in Hoffman's original question, which deals only with limit points of the spectral radius of graphs. We recall from the work of V. Nikiforov that for an undirected graph G the matrix

$$A_\alpha(G) := \alpha D(G) + (1-\alpha)A(G),$$

for $0 \leq \alpha \leq 1$. We study the A_α version of Shearer's results.

Our main result is that for any $\alpha \in [0, 1/2)$ there exists a positive number $\tau_2(\alpha) > 2$ such that any value $\lambda > \tau_2(\alpha)$ is an A_α-limit point. Additionally, we study, for small values of α, the existence of intervals $[\tau_1(\alpha), \tau_1'(\alpha))$ for which all numbers are also A_α-limit points.