A subgroup property P is an *embedding property* if in any group G all images under automorphisms of G of P-subgroups likewise have the property P. In the context of the project “Large Soluble Groups”, the notion of a “large group” can be formalized in the following way:

Let *U* be an universe of groups, and let *X* be a non-empty subclass of *U*. Then *X* is said to be a class of *large groups* in the universe *U* with respect to a given embedding property *P* if it satisfies the following conditions:

- If an U-group *G* contains a subgroup isomorphic to a group in *X*, then *G* belongs to *X*;
- If *G* is any *X*-group and *N* is a normal subgroup of *G*, then at least one of the groups *N* and *G/N* belongs to *X*;
- No cyclic groups belong to *X*;
- If *G* is an *X*-group in which every *X*-subgroup has the property *P*, then all subgroups of *G* have the property *P*.

A proper subclass *Y* of *U* consists of *small groups* in the universe *U* with respect to the property *P* if the complementary set of Y in U is a class of large groups with respect to the same property.

The idea of these definitions arises from some results obtained by several authors who have worked on the class of groups of infinite rank. A group G is said to have finite (Prufer) rank if there exists a positive integer r such that all finitely generated subgroups of G can be generated by at most r elements; otherwise, if such r does not exist, the group is said to have infinite rank. It is not difficult to see that in some universes of (generalized) soluble groups, if a group G has infinite rank, then it must be rich in subgroups of infinite rank. Moreover, in recent years, a series of relevant papers has been published which show that the subgroups of infinite rank of a group of infinite rank have the power to influence the structure of the whole group and to force also the behaviour of the "small" subgroups of G (i.e. the subgroups of finite rank). In fact, it has been proved that, for some choices of group theoretical properties X, if G is a group in which all subgroups of infinite rank satisfy the property X, then the same happens also to the subgroups of finite rank. Among these results it is of special relevance one due to J. Evans and Y. Kim, which states that if all subgroups of infinite rank of a locally soluble group G of infinite rank are normal, then every subgroup of G is normal. In our language, the theorem of Evans and Kim just says that groups of infinite rank form a class of large groups with respect to normality in the universe of locally soluble groups. Other authors (among which M.R. Dixon, J. Evans, Z.Y. Karatas, L.A. Kurdachenko, N.N. Semko, H. Smith) have proved that the structure of a (generalized) soluble group is influenced also by the behaviour of its subgroups of infinite rank also with respect to some natural generalizations of normality as, for example, subnormality or permutability.

Moreover, also some absolute properties have a good behaviour with respect to subgroups of infinite rank. An interesting result in this context is due to M.R. Dixon, M.J. Evans and H. Smith, who have proved that if c is a positive integer, and G is a locally soluble group of infinite rank whose proper subgroups of infinite rank are nilpotent with class at most c, then G itself is nilpotent with class at most c. The same authors have also proved that if k is a positive integer, and G is a soluble group of infinite rank whose proper subgroups of infinite rank have derived length at most k, then G itself has derived length at most k (this means, in particular, that if all proper subgroups of infinite rank are abelian, then G is abelian). Maria De Falco, jointly with other authors, have worked on the same type of problem for different choices of the absolute property. In particular, jointly with Francesco de Giovanni, Carmela Musella and Nadir Trabelsi she has proved that if X is any class of groups with some natural propertis of closure, and G is a locally (soluble-by-finite) groups in which all proper subgroups of infinite rank belong to X, then some informations can be obtained on the structure of G: they have obtained some results and developed some techniques which can be used as a tool in different situations. Using this machinery, it has been proved by Maria De Falco, jointly with Francesco de Giovanni, Carmela Musella, Yaroslav P. Sysak, Nadir Trabelsi, that, for many natural choices of the class X, whenever G is a (generalized) soluble group of infinite rank in which all proper subgroups of infinite rank are X-groups, then G itself is an X-group.

The project “Large soluble groups” is based on the persuasion that it is possible to develop the work about the groups of infinite rank, showing that all the natural generalizations of normality pass from the subgroups of infinite rank to the ones of finite rank. In particular, we call the following properties “*embedding properties of normal type”* (because they are the natural generalizations of normality): normality, subnormality, permutability, Neumann’s properties (which consider normality with the obstruction of a finite section), pronormality (which relates to conjugacy problems derived from the classical Sylow’s theorem), and modularity (i.e. the property of being the image of a normal subgroup under an isomorphism between subgroup lattices). The main goal of the project is to prove that (at least within the universe of soluble groups) the class of groups of infinite rank is a class of large groups with respect to all embedding properties of normal type.