On the equivalence of linear sets of rank \(n\) in \(\text{PG}(1, q^n)\)

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Let \(\Lambda = \text{PG}(W, \mathbb{F}_q) = \text{PG}(r-1, q^t)\), where \(W\) is a vector space of dimension \(r\) over \(\mathbb{F}_q\). A point set \(L\) of \(\Lambda\) is said to be an \(\mathbb{F}_q\)-linear set of \(\Lambda\) of rank \(n\) if it is defined by the non-zero vectors of an \(n\)-dimensional \(\mathbb{F}_q\)-vector subspace \(U\) of \(W\), i.e.

\[
L_U = \{\langle u \rangle_{\mathbb{F}_q} : u \in U \setminus \{0\}\}.
\]

One of the most natural questions about linear sets is their equivalence. The linear sets \(L_U\) and \(L_V\) of \(\text{PG}(r-1, q^t)\) are said to be equivalent if there is an element \(\phi\) in \(\text{PGL}(r, q^t)\) such that \(L_{\phi U} = L_V\). In the applications it is crucial to have methods to decide whether two linear sets are equivalent or not. An efficient way to do this is to consider linear sets as projections of subgeometries which is a result of Lunardon and Polverino. Csajbók and Zanella gave a characterization of those linear sets for which the question of equivalence can be translated to the study of the orbits of the stabilizer of a subgeometry on subspaces. They proved that these are exactly those linear sets \(L_U\) which satisfy the condition that \(L_U = L_V\) implies the existence of a non-singular \(\mathbb{F}_q\)-semilinear map of \(W\) between \(U\) and \(V\). We will call such linear sets \textit{simple}. Until recently only one family of non-simple linear sets was known, the linear sets of pseudoregulus type in \(\text{PG}(1, q^n)\).

In this talk we present further examples of non-simple linear sets of rank \(n\) of \(\text{PG}(1, q^n)\) for \(n \geq 5\), and we show that linear sets of rank \(n\) of \(\text{PG}(1, q^n)\) are simple for \(n \leq 4\). Contrary to what we expected, simple linear sets are harder to find. We give some evidence for this claim. Simple linear sets of rank \(n\) of \(\text{PG}(1, q^n)\) for each \(n\) will also be presented.