

# Enlargements and autoconjugate representations of maximally monotone operators

M. A. Théra<sup>1</sup>

## Abstract:

The aim of this talk consists in presenting new results on enlargements as well as on autoconjugate representations of maximally monotone operators. More precisely, let  $X$  be a real Banach space with continuous dual  $X^*$ . We consider a generalized equation governed by a maximally monotone operator  $T$  from  $X$  into the subsets of  $X^*$ , that is the problem of finding  $x \in X$  such that  $0 \in T(x)$ . This model has been extensively used as a mathematical formulation of fundamental problems in optimization and fixed point theory.

Solving the previous inclusion is tantamount to finding a point of the form  $(x, 0)$  in the graph of  $T$ . When  $T$  is set-valued, the problem could be ill-behaved, making the required computations hard. Enlargements of  $T$  are point-to-set mappings (the terms set-valued mapping and multifunction are also used) which have a graph larger than the graph of  $T$ . These mappings, however, have better continuity properties than  $T$  itself. Moreover, they stay “close” to  $T$ , so they allow to define perturbations of the initial problem, without losing information on  $T$ . In this way, we can define well-behaved approximations of the initial problem, which (i) are numerically more robust, and (ii) whose solutions approximate accurately the solutions of  $0 \in T(x)$ . The use of enlargements in the study of such a problem has been a fruitful approach, from both practical and theoretical reasons.

One key ingredient for this study is the (variational) representation of operators as introduced independently by Fitzpatrick et Martinez-Legaz & Théra.

It is nowadays well established that:

- each representable operator is monotone;
- each monotone operator is not necessarily representable;
- each maximally monotone operator is representable;
- a representable operator is not necessarily maximally monotone.

In other words, the family of representable operators forms a class strictly included in between monotone operators and maximally monotone operators.

A typical example of representable operator is given by the subdifferential of a lower semicontinuous proper function  $f$ . It can be observed that it is represented by the Fenchel-Young mapping :  $g(x, x^*) = f(x) + f^*(x^*)$  associated to  $f$ , where  $f^*$  stands for the Fenchel conjugate of  $f$ . It should be noticed that this representation is auto-conjugate in a sense which will be defined during the lecture. We will also give some recent developments on auto-conjugate representations of maximally monotone operators.

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<sup>1</sup> Université de Limoges and Federation University, Ballarat  
*micel.thera@unilim.fr*