# PRIN 2012 "Geometric Structures, Combinatorics and their Applications"

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## 1 Short Description of the Project

The main objective of this Project is to develop and promote basic research in the framework of Incidence Geometries, Galois Geometries and Combinatorics, both through active investigation performed by the involved researchers on subjects of great up-to-date scientific interest and through the organisation of national and international Conferences, Workshops and specialised Schools. Further, special computer algebra software packages will be used, and new sub-packages will be written, for studying finite structures. As for applications, joint research will be developed both with researchers, also outside of the academic world, and with public and private research institutions.

The main themes are listed below which are relevant to this Project as well as those problems to whose solution significant contributions will hopefully be made.

**[A] Finite Geometries and Incidence Geometries**

[A1] New constructions, characterisations of geometric structures in finite projective planes and higher dimensional spaces, such as arcs, caps, blocking sets, linear sets, spreads.

[A2] Geometric and combinatorial characterisations of significant algebraic curves and surfaces in finite projective spaces, search for bounds on the number of their rational points, and possible links with linear codes.

[A3] Construction and classification of Geometries belonging to an assigned Tits-Buekenhout diagram.

[A4] Characterisation of incidence geometries (such as, for instance, linear and semilinear spaces, projective planes, Benz planes, absolute geometries), endowed with a set of automorphisms having special properties, such as transitivity, forming a group, etc.

[A5] Embedding of special classes of incidence geometries in projective geometries.

[A6] Classification by axioms of some linear and semilinear spaces.

[A7] Characterisation of incidence geometries coming from significant algebraic varieties.

[A8] New constructions of ovoids and spreads in classical polar spaces.

[A9] Construction of new Kerdock and Preparata codes

**[B] Design Theory and Graph Theory**

[B1] The existence as well as to construct some optical orthogonal codes with arbitrary Hamming weight, auto-correlation parameter 2 and cross-correlation parameter 1.

[B2] New constructions, geometric and/or combinatorial characterisations and investigation of authomorphism groups of special point configurations such as arcs, blocking sets and unitals.

[B3] Investigation of new nestings and new metamorphoses of G-designs.

[B4] Possibilities of solving the dynamic grooming problem for some networks.

[B5] Investigation of Voloshin colourings for Steiner systems of index ¿ 1 and improvements, if possible, of the known results for systems of index 1. Voloshin colourings of G-designs.

[B6] Characterisation of graphs associated with incidence geometries and/or algebraic structures.

[B7] Spectral Determination or Characterization of graph.

[B8] Investigation of minimum covering and maximum packing problems for a complete graph by copies of assigned graphs.

[B9] Embedding of graphs in surfaces.

## 2 Performed Activities

It is well known that any finite geometry may be investigated via its incidence graph, that Tits-Buekenhout geometries may be defined as special multipartite graphs, that every design defines a linear code by its incidence matrix and, finally, that some techniques of algebraic combinatorics use methods coming from graph theory. The fact that methods and results are in a sense complementary, as mentioned above, provides a real motivation for the cultural and scientific need which lead to bring together the units participating in this Project. The central objective of this programme consists in further developing investigations in the fields of geometric structures and combinatorics, by taking into account all their features (combinatorial, geometrical, algebraic, computational, algorithmic, etc.). Both the results achieved in such fields during the recent years and the well proved scientific expertise of each research unit will form the basis to develop new techniques which, hopefully, will allow to tackle open problems in those theories the proposed researches deal with.

Among the supported activities which seem to be fundamental to the success of this Project, the following are mentioned:

(a) The organisation of local seminars by each local unit.

(b) The strenghtening of the national seminar Geometric Structures, Combinatorics and their Applications held at the Mathematics Department of the Universit di Napoli.

(c) The publication of volumes, internal reports, preprints, lecture notes and whatever might turn out useful to spread and coordinate results and improvements related to all researches in progress.

(d) The management of the project webpage, acquainting the national and international research community with the activities of the local units and with the obtained results.

Organisation tasks are assigned as follows:

1) besides the usual tasks due to coordinating this Project, the local unit of Naples will organise in May-June 2014 the international conference Combinatorics 2014;

2) the local unit of Padova will organise a workshop on the recent developments of the theory of finite semifields and will look after the project web page at the address http://147.162.25.187/;

3) the local unit of Catania will support the unit of Modena in the organisation of a workshop on the recent developments of Design theory and Graph theory;

4) the local unit of Perugia will support the unit of Potenza in the organisation of the workshop on the recent developments of Galois Geometries;

5) the local unit of Milano will organise in 2016 the final project meeting in order to check the results obtained by the researchers participating in this Project;

6) the local unit of Modena will organise a workshop on the recent developments of Design theory and Graph theory;

7) the local unit of Brescia will support the unit of Potenza in the organisation of the workshop on the recent developments of Galois Geometries;

8) the local unit of Potenza will organise, in Summer 2015, a workshop on the recent developments of Galois Geometries.

The organisation of the conference "Combinatorics 2016" will be appointed to a unit to be decided at a later stage.

# 3 The Local Unit of D.M.A.

## 3.1 Members

**Guglielmo Lunardon** (Full Professor),
**Domenico Olanda** (Full Professor),
**Laura Bader** (Associated Professor),
**Francesco Belardo** (Associated Professor),
**Nicola Durante** (Associated Professor),
**Rocco Trombetti** (Associated Professor),
**Giorgio Donati** (Ricercatore),
**Yue Zhou** (Post Doc Fellow),
**Francesco Mazzocca** (Full Professor, S.U.N.),
**Olga Polverino** (Full Professor, S.U.N.),
**Giuseppe Marino** (Associated Professor, S.U.N.),
**Ilaria Cardinali** (Ricercatore, University of Siena),

## 3.2 The Main Research Themes

The main research themes are planned to be: Finite Geometries and Translation Planes, and Incidence Geometries and Linear Spaces.

Some details on the points above follow.

a) **Finite Geometries and Translation Planes**

We recall that any semifield S of dimension two over its left nucleus Nl=GF(q) may be constructed via a GF(s)-linear set L of rank 2r, with q=sr , skew to a hyperbolic quadric Q+(3,q). The subfield GF(s) of Nl is the centre of the semifield. In the recent paper G. Marino, O. Polverino, R. Trombetti: On Fq-linear sets of PG(3,q3) and semifields, J. Comb. Theory A, 2007 it was proved that semifields of dimension two over their left nucleus and of dimension six over their centre can be divided into five families for some of which no example is known. To continue the already started research, the attempt will be made to construct new classes of semifields of dimension two over their left nucleus and dimension greater than six over their centre, within the above mentioned families. Using again linear sets, new infinite families are aimed to be constructed of semifields of dimension greater than four over their centers.

In G. Lunardon: Translation ovoids, J. Geom. 76, 2003, by generalising a construction by J.A. Thas, a new semifield ST is associated with a semifield S of dimension two over its left nucleus and ST has the same dimension over its left nucleus. In all known examples the middle and right nucleus of ST have the same order of the middle and right nucleus of S, respectively. We want to prove that this property is independent of the choice of the semifield S. In G. Lunardon: Blocking sets and semifields, J. Comb. Theory A 113, 2006 it is proved that an indicator set associated with a semifield is a minimal small blocking set of Redi type in PG(2,q2) which is skew to a Baer subline. Using the link between translation ovoids of H(3,q2) and semifields described in the above mentioned paper, it is planned to characterise such blocking sets. A spread of PG(2n-1,q) is symplectic if its elements are totally isotropic with respect to a symplectic polarity. It was recently shown (G. Lunardon : Symplectic spreads and finite semifields, Des. Codes Crypt. 44, 2007) that if the semifield S defines a symplectic spread, then its right and its middle nucleus coincide and form the centre of S. A cubical array defines a 1-1 correspondence between symplectic semifields and commutative semifields; since the latter are rare, as quite recently W.M. Kantor observed, the problem of constructing possibly new examples is specially interesting, also because symplectic spreads are related to the theory of Z4-linear codes. Let W(5,q) be the polar space associated with a symplectic polarity of PG(5,q), q even. In I. Cardinali, G. Lunardon: A geometric description of the spin-embedding of symplectic dual polar spaces of rank 3, J. Comb. Theory A, in press), an embedding is constructed in Q+(7,q) of the dual space of W(5,q); using this embedding, we want to prove that any symplectic spread is transformed into an ovoid of Q+(7,q) and using such a result we wish to construct new examples of ovoids and symplectic spreads.

The characterisation of special sets in finite projective spaces, such as, for instance, conics, quadrics and intersections of classical varieties, is a traditional problem in Finite Geometries. In particular, we wish to construct new two character sets and study the intersections of Hermitian varieties and subgeometries in a projective space.

b) **Incidence Geometries and Linear Spaces** As to Incidence Geometries, in order to generalise a recent result on sets which block the exterior lines to a conic (cf. A. Aguglia, G. Korchmaros: Blocking sets of nonsecant lines to a conic in PG(2,q), q odd, J. Comb. Des., in press), we want to characterise the sets of lines which meet all exterior lines to a non-singular quadric in PG(n,q).

One of the traditional themes in the theory of Linear Spaces consists in estabilishing when they can be embedded in a finite projective space; in spite of the many already existing results, this problem is not completely solved. Therefore, new characterisations of embeddable linear spaces will be looked for.

Finally, some members of this Research Unit are interested in some applications to other branches of Mathematics. A traditional application is the study of classical groups and the incidence geometries associated with them; to this aim, a deeper investigation of the geometric properties of such groups will be undertaken both in the finite and the infinite case.

This Local Unit will continue to organise the already well appreciated Seminar Geometric Structures, Combinatorics and their Applications and perform the traditional coordinating tasks.